

Mathematical Modelling of Aluminum Reduction Cell Potshell Deformation

Marc Dupuis

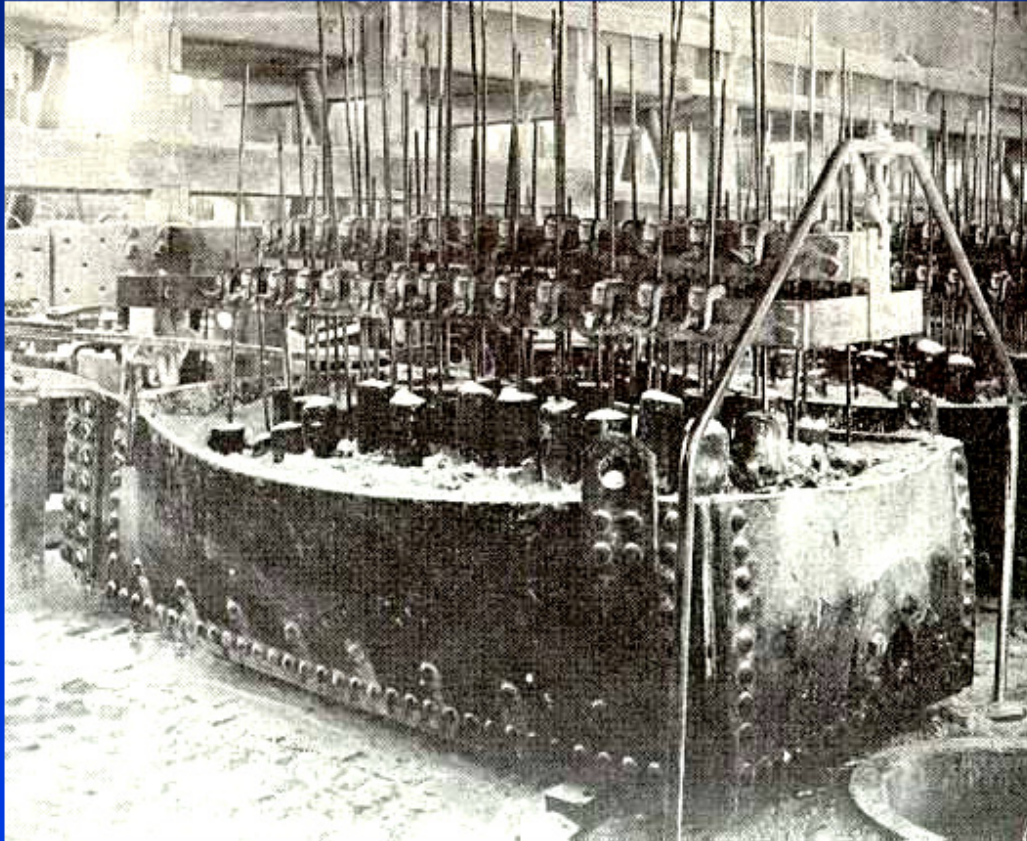
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Plan of the Presentation

- **Introduction**
 - Dewing stress-strain relationship of the sodium swelling
 - Dewing sodium diffusion coefficient
 - Historical models development background
- **“Empty Shell” Potshell Model**
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 - Plastic mode
- **“Almost Empty Shell” Potshell Model**
 - Elastic mode
 - Plastic mode
- **“Half Empty Shell” Potshell Model**
 - Elastic mode
 - Plastic mode
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Introduction

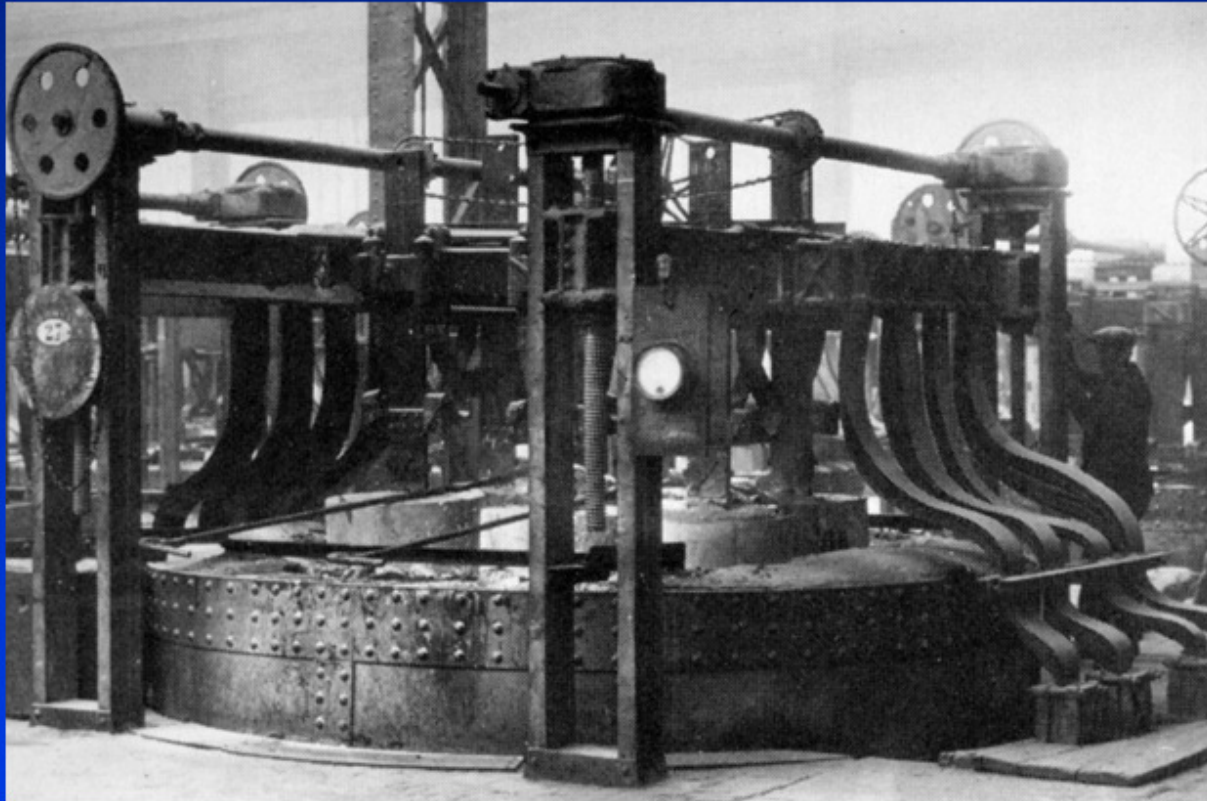


A well-designed shell is supposed to withstand the internal forces induced by thermal and chemical changes in the reduction cell.

The thermal changes are quite straightforward to assess if a complementary thermo-electric model is available .

The chemical changes is the consequence of sodium penetration into the carbon cathode blocks. This sodium penetration makes the carbon swell and hence, the cathode blocks expand chemically.

Introduction



One quite acceptable technical solution:

If the potshell want to be round, why not build it round in the first place?

Dewing Stress-Strain Relationship of the Sodium Swelling

The physics of that chemical expansion is not well understood. The key references on that subject are Rapoport and Dewing.

According to Dewing, the stress-strain relationship of the sodium swelling of the carbon is:

$$\varepsilon = \varepsilon_0 10^{-k\sigma}$$

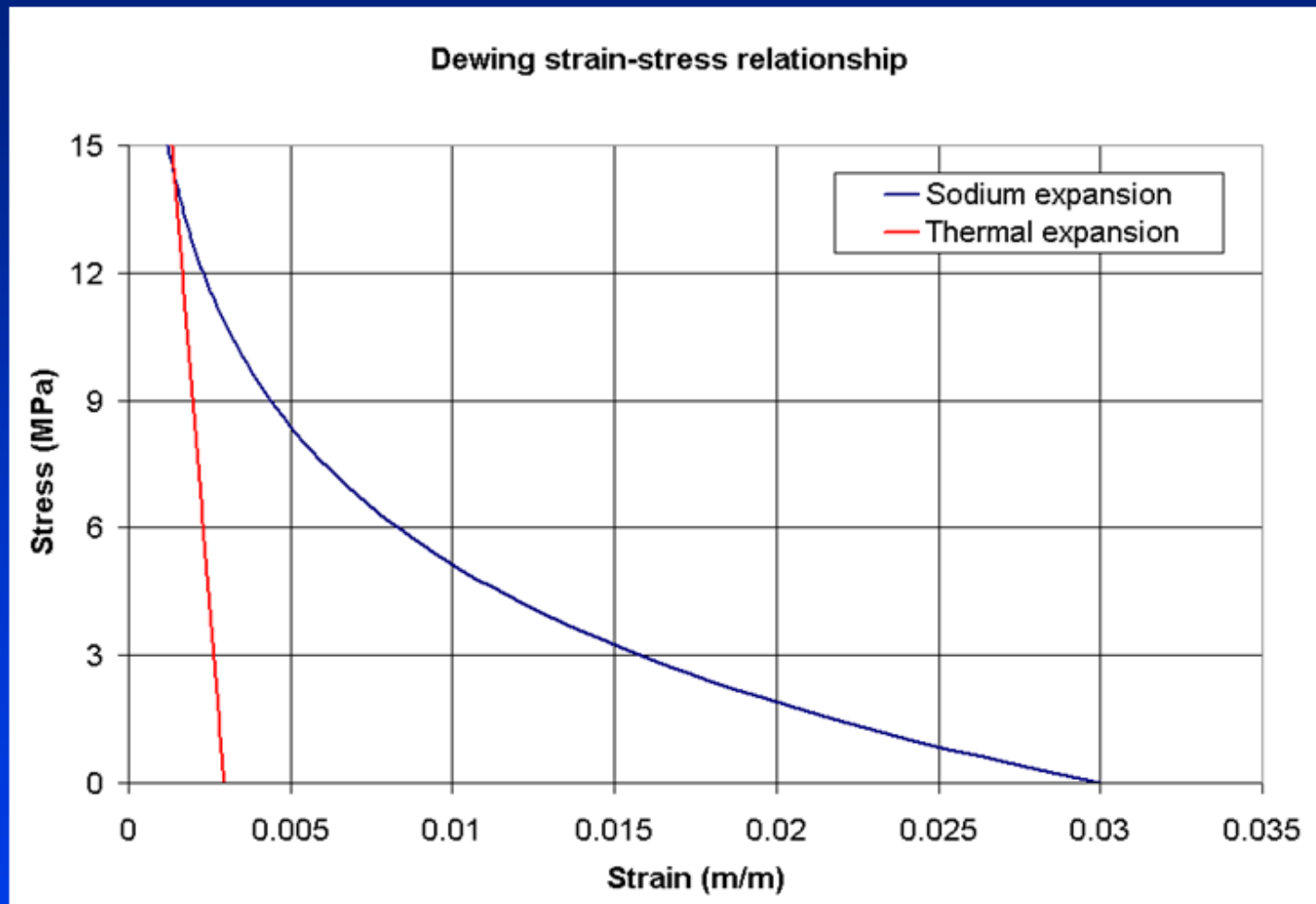
where: ε is the carbon strain at equilibrium

ε_0 is the carbon free expansion strain

σ is the compressive stress in the carbon

k is a constant

Dewing Stress-Strain Relationship of the Sodium Swelling

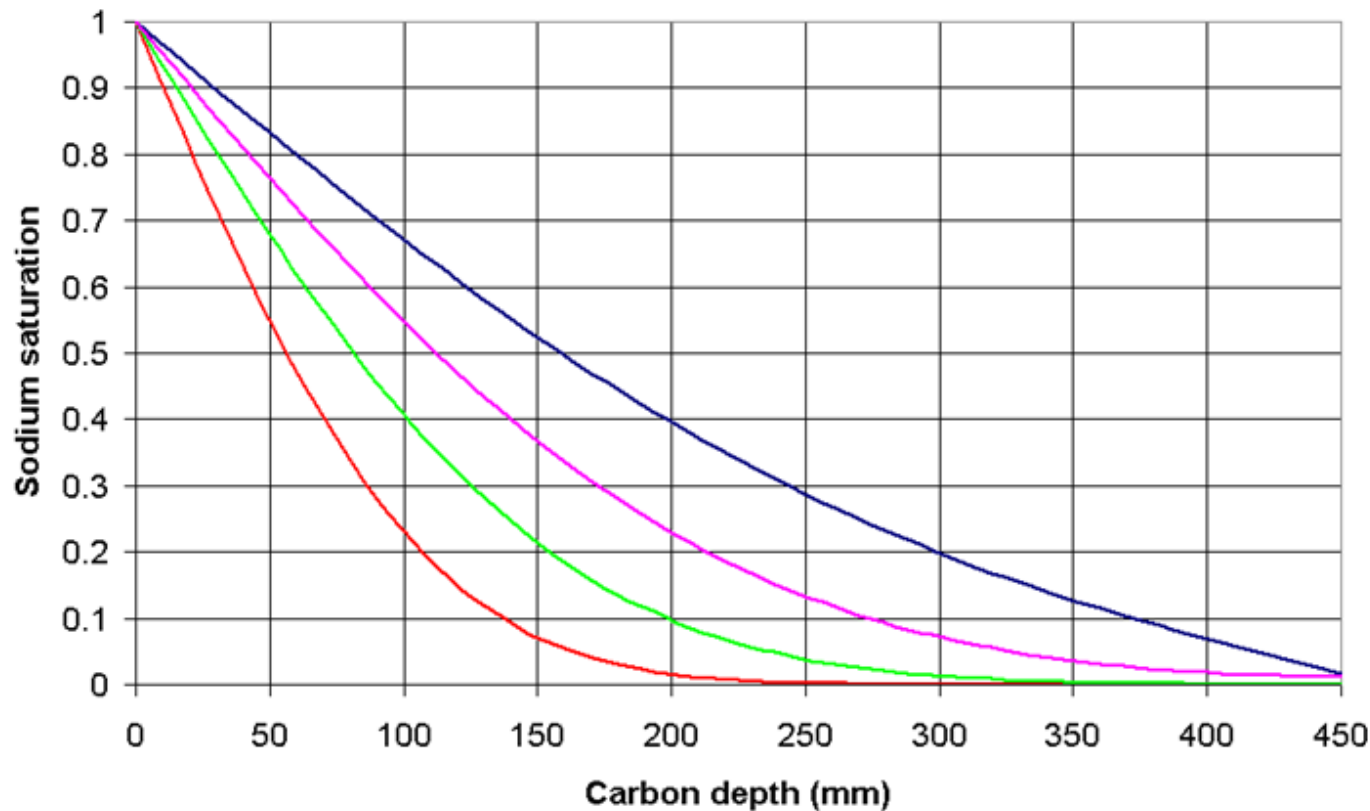


Dewing has estimated k to be equal to $6.4E-4$ when expressed in PSI, which corresponds to 0.092825 when expressed in MPa.

On the other hand, the carbon free expansion strain varies a lot depending on the carbon grade and quality. 3% is a typical value for 20% semi-graphitic cathode grade, which is about ten times more than the thermal expansion.

Dewing Sodium Diffusion Coefficient

Sodium saturation after 10, 20, 40 and 80 days



Dewing has also established the sodium diffusion coefficient to be equal to $4E-5 \text{ cm}^2/\text{s}$, which corresponds to $3.456E-4 \text{ m}^2/\text{day}$.

Corresponding sodium saturation for a typical 45 cm thick cathode block for the full block thickness after 10, 20, 40 and 80 days of operation.

Dewing Sodium Diffusion Coefficient

Curves presented in previous slide can be obtained by numerically solving the diffusion partial differential equation:

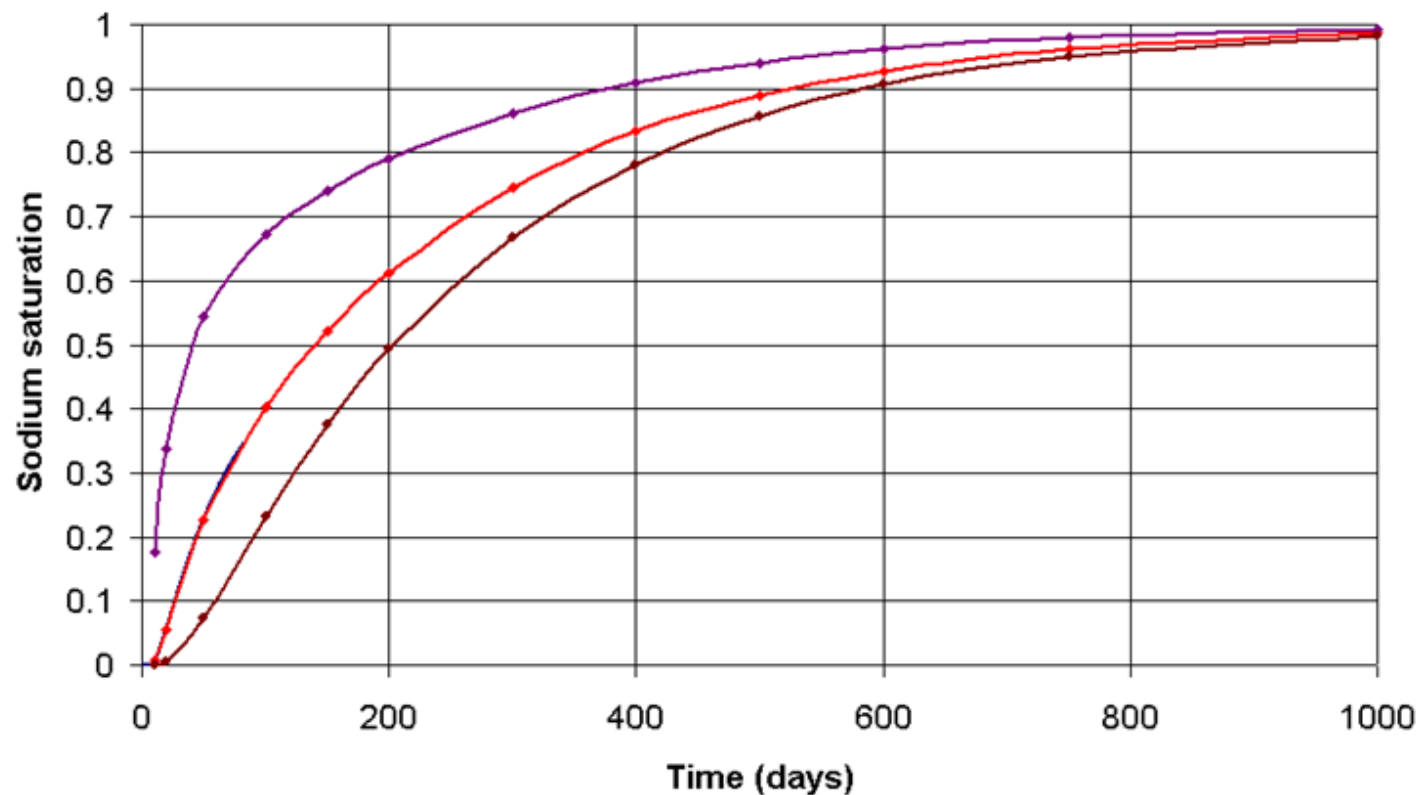
$$\frac{\delta^2 C}{\delta x^2} = \frac{1}{D} \frac{\delta C}{\delta t}$$

Alternatively, it is possible to use the analytical solution of that diffusion equation also presented by Dewing:

$$C = C_0 \left\{ 1 - \frac{4}{\pi} \sum_{n=0}^{\infty} \frac{1}{(2n+1)} \sin \left[\frac{(2n+1)\pi x}{2h} \right] \exp \left[- \left(\frac{(2n+1)\pi}{2h} \right)^2 Dt \right] \right\}$$

Dewing Sodium Diffusion Coefficient

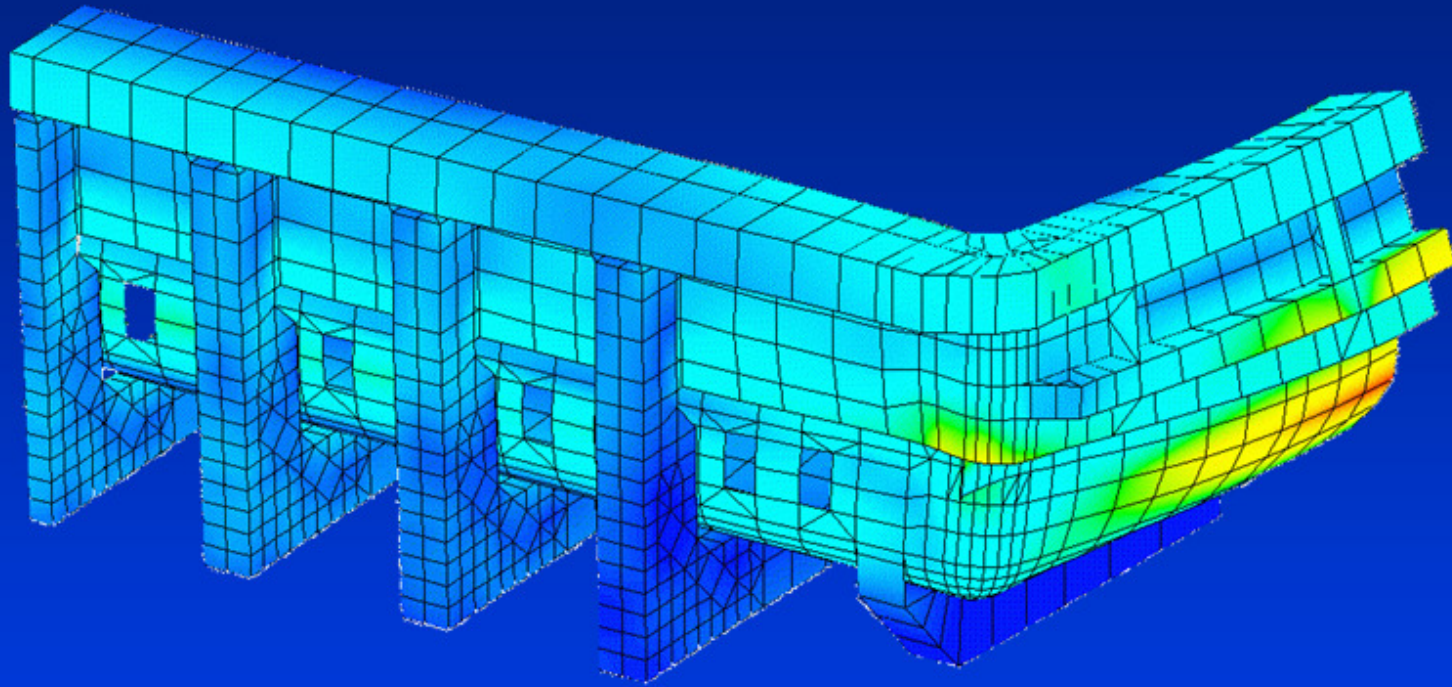
Sodium saturation in cathode block as function of time and depth at 1/4 1/2 and 3/4 of the cathode block thickness



Corresponding sodium saturation for the same typical 45 cm thick cathode block, this time at $x = 1/4, 1/2$ and $3/4$ of the total block thickness as function of time as computed by that analytical equation.

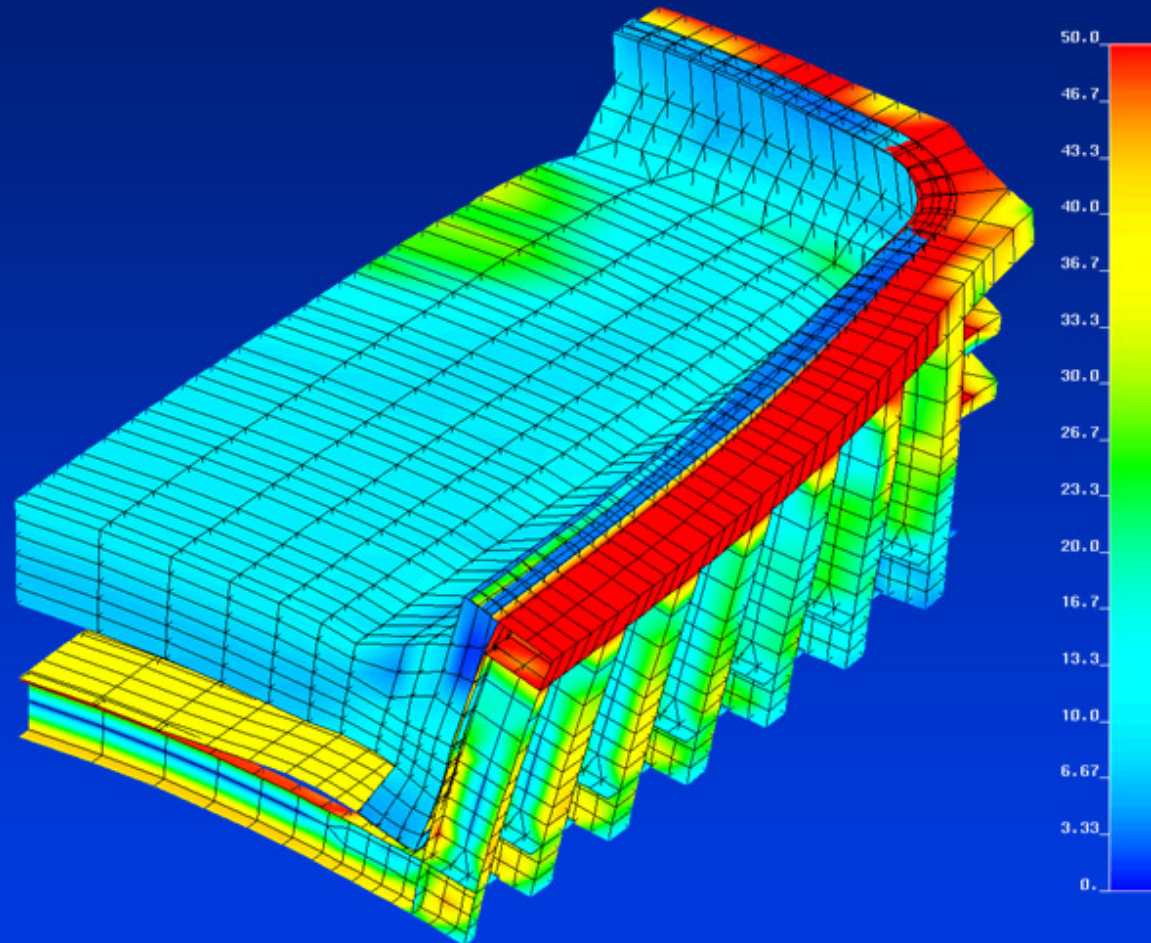
It takes about 80 days for the sodium diffusion front to reach the bottom of the cathode block and about 1000 days for the cathode block to be fully saturated in sodium from top to bottom.

Historical Models Development Background



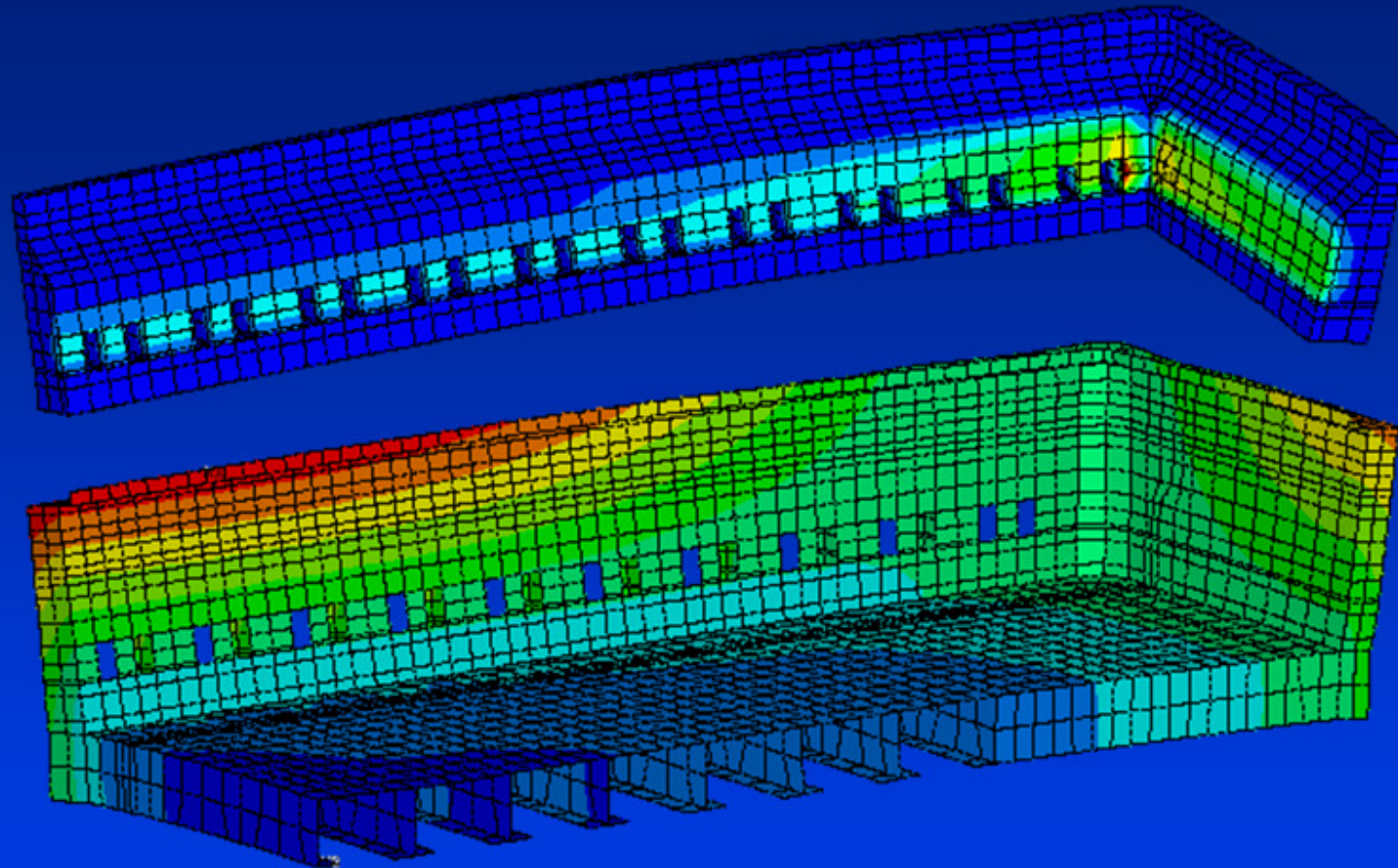
First “Empty shell” potshell model using plastic mode developed in 1988 and presented at the TMS in 1991

Historical Models Development Background



First “Half Empty shell” potshell model developed in 1989 and presented at a CRAY Supercomputing Symposium in 1990

Historical Models Development Background



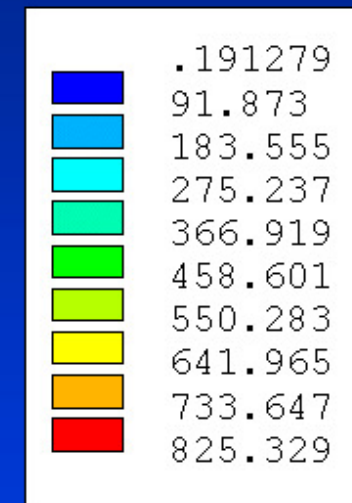
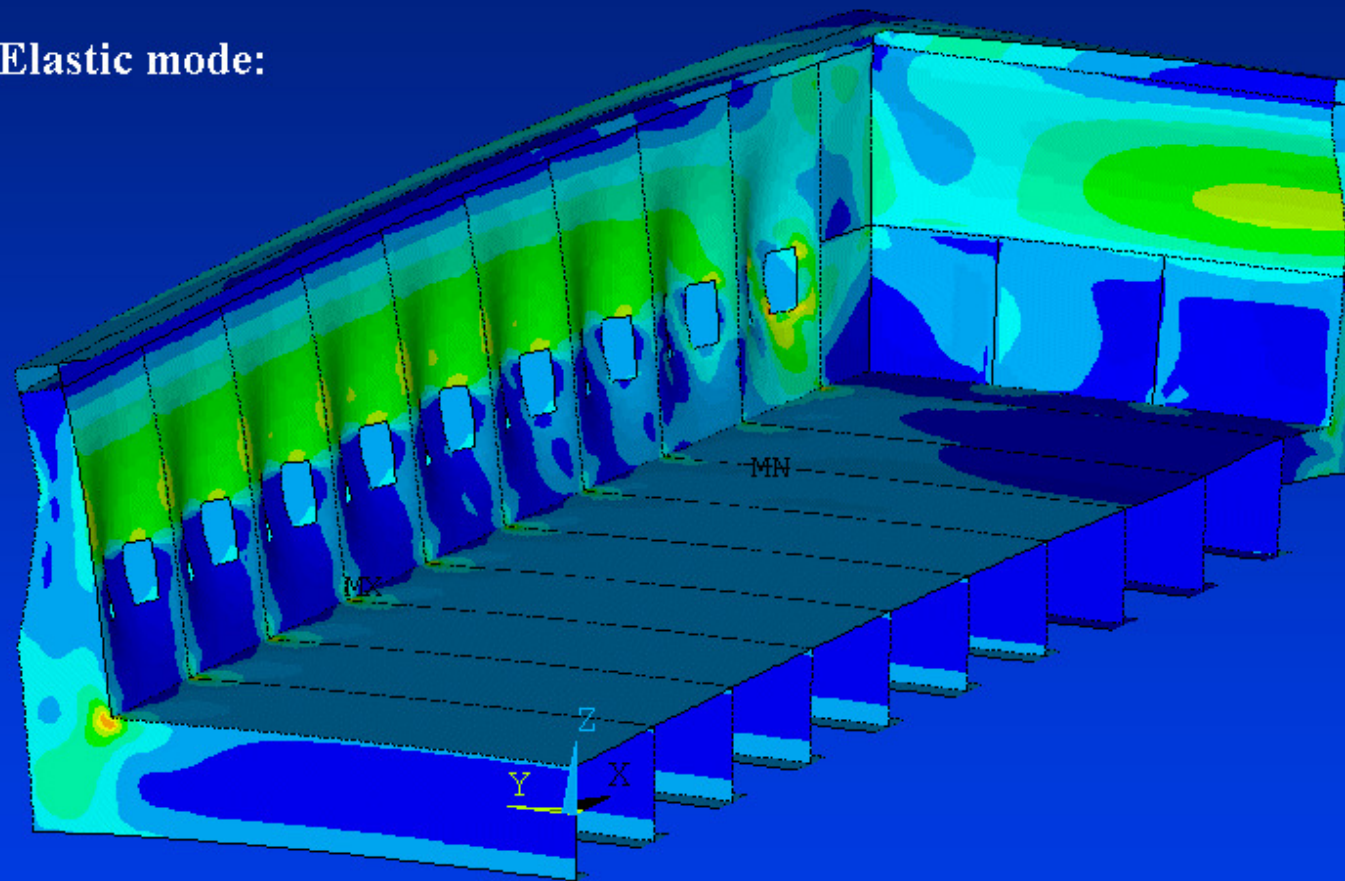
First “Almost Empty shell” quarter potshell model developed in 1992 and presented at CIM in 1993

“Empty Shell” Potshell Model

- Based on the usage of the quadrilateral Finite Strain shell element (SHELL181) in the commercial code ANSYS®.
- Temperature distribution obtained from the full cell quarter thermo-electric model applied as a body load to the entire potshell structure.
- Internal pressure (or forces) loading scheme based on the cathode block size and position relative to the potshell. That loading scheme is at best semi-empirical and is typically considered as a trade secret.
- Elastic mode: solving the mechanical problem only by considering only the elastic properties of the potshell steel.
- Plastic mode: solving the temperature dependent isotropic hardening von Mises plasticity behavior of the potshell steel structure using the MISO non-linear hardening option in ANSYS®.

“Empty Shell” Potshell Model

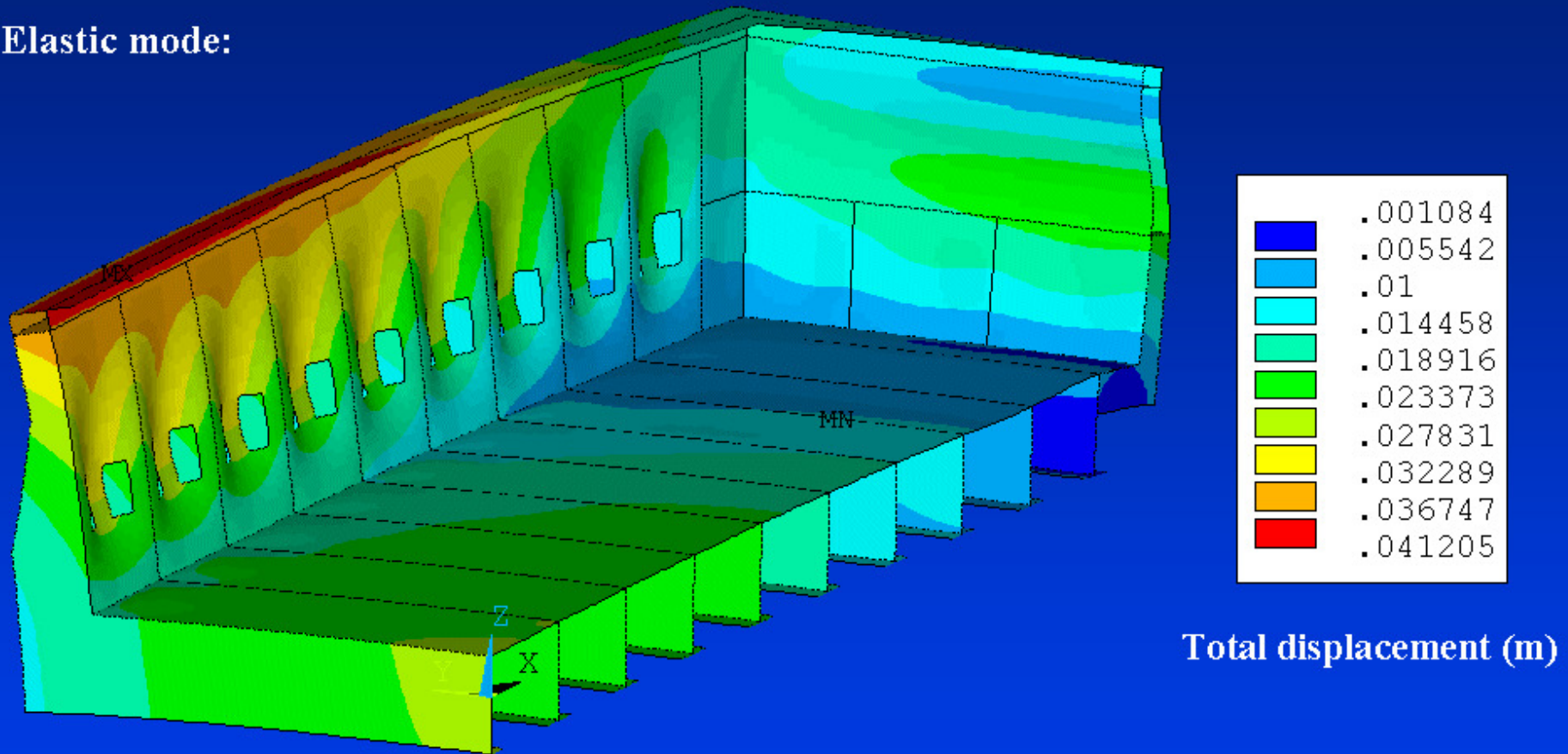
Elastic mode:



von Mises Stress (MPa)

“Empty Shell” Potshell Model

Elastic mode:

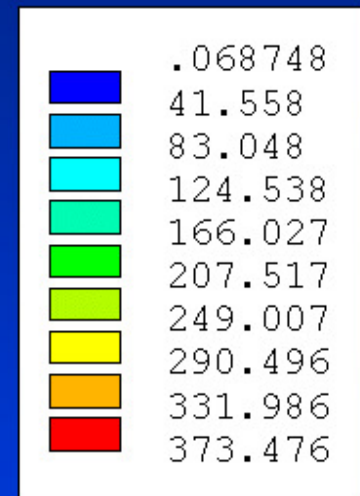
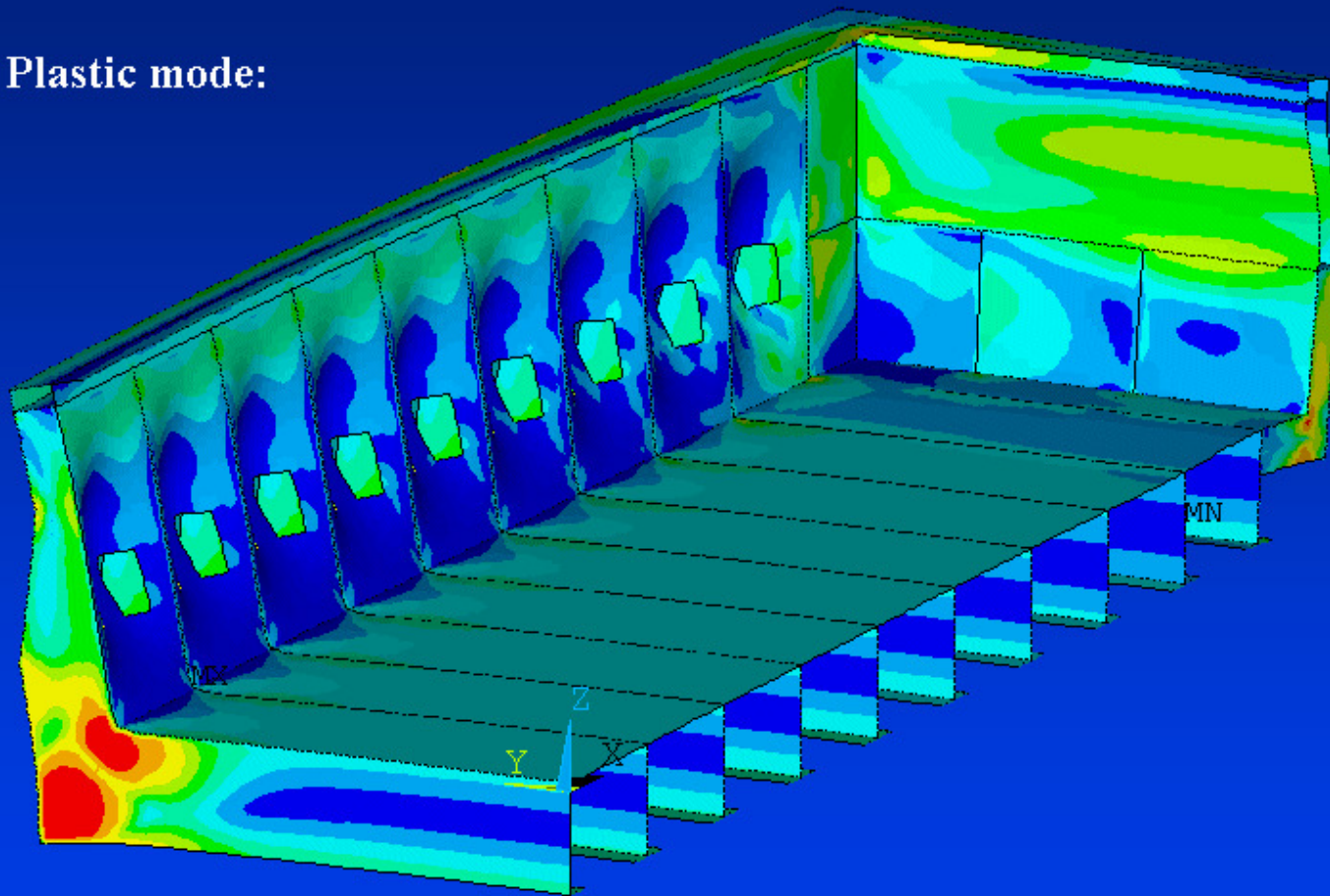


Took only 150 CPU seconds to solve on a 64 bits dual core Intel Centrino T9300
Dell Precision M6300 portable computer running ANSYS® 11.0 version.

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“Empty Shell” Potshell Model

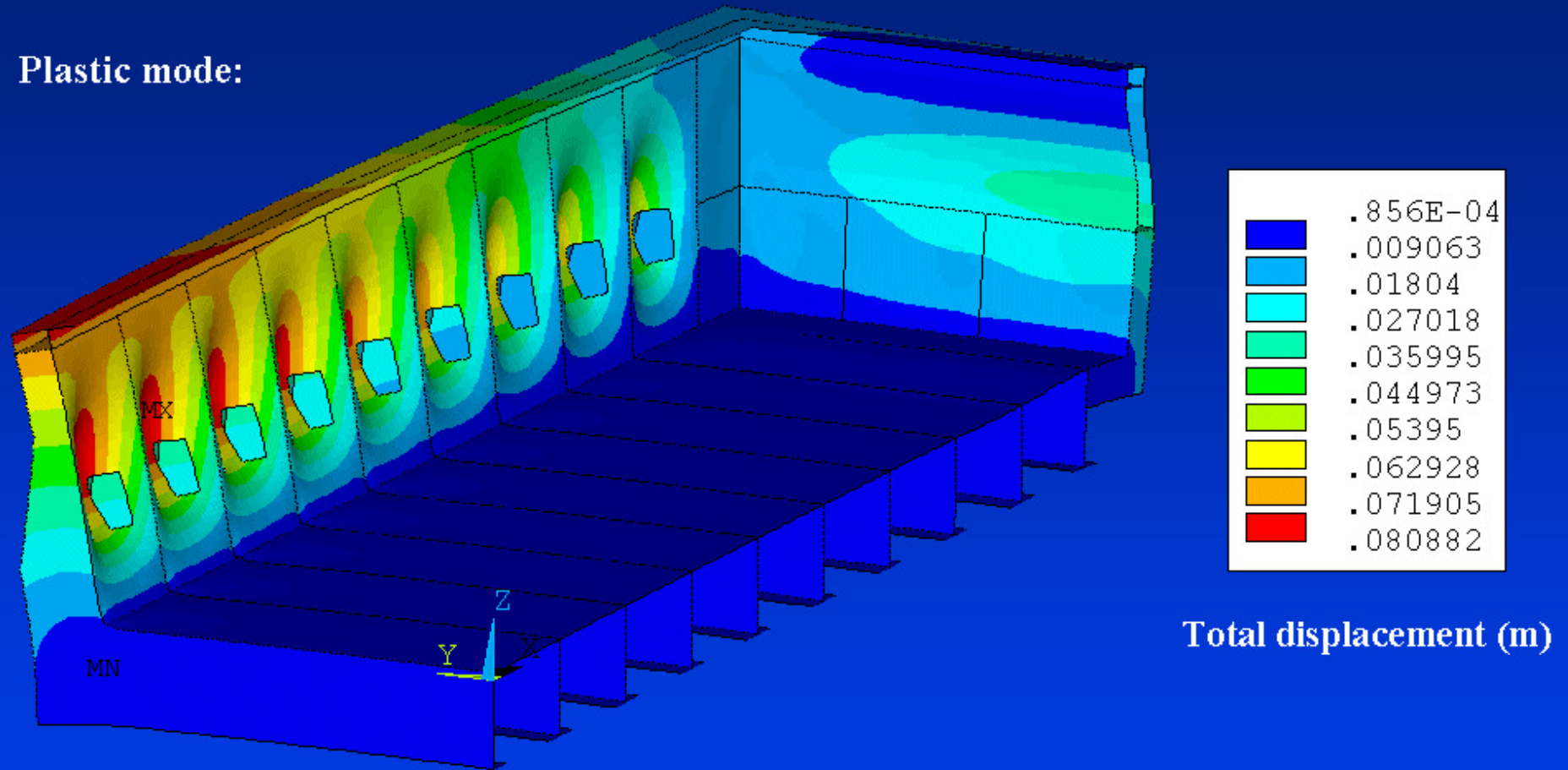
Plastic mode:



von Mises Stress (MPa)

“Empty Shell” Potshell Model

Plastic mode:



Took 11 times more CPU to solve than in elastic mode, but still this is only 1706 CPU seconds or 28 CPU minutes.

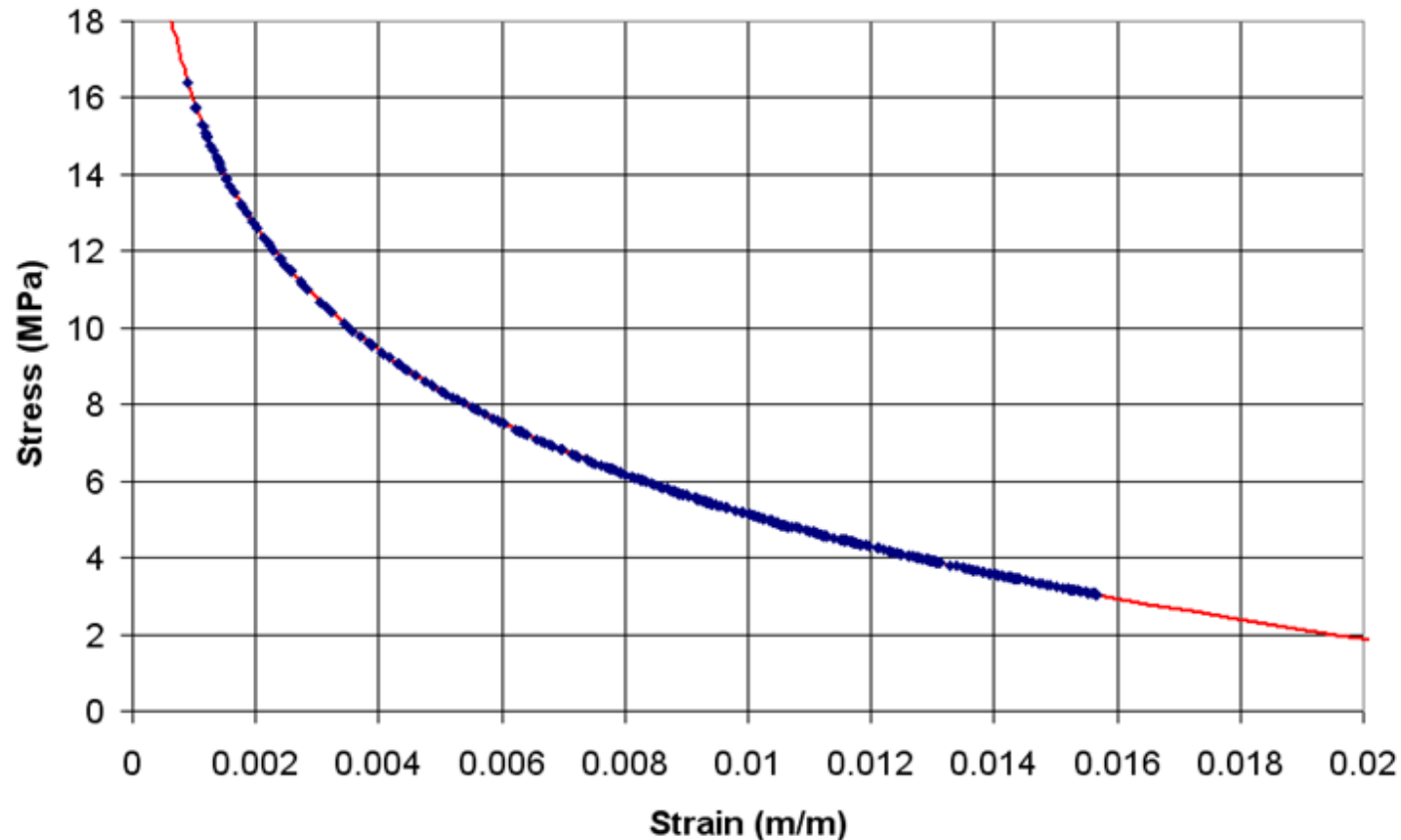
“Almost Empty Shell” Potshell Model

- Based on the usage of the quadrilateral Finite Strain shell element (SHELL181) in the commercial code ANSYS®.
- Temperature distribution obtained from the full cell quarter thermo-electric model applied as a body load to the entire potshell structure.
- Adding the lining geometry between the potshell walls and the cathode blocks all around the potshell and applying a pressure loading as boundary condition at the carbon block/side lining interface that is lying on the Dewing strain-stress relationship.
- Elastic mode: solving the mechanical problem only by considering only the elastic properties of the potshell steel.
- Plastic mode: solving the temperature dependent isotropic hardening von Mises plasticity behavior of the potshell steel structure using the MISO non-linear hardening option in ANSYS®.

“Almost Empty Shell” Potshell Model

Individual element face strain-stress location on the Dewing curve

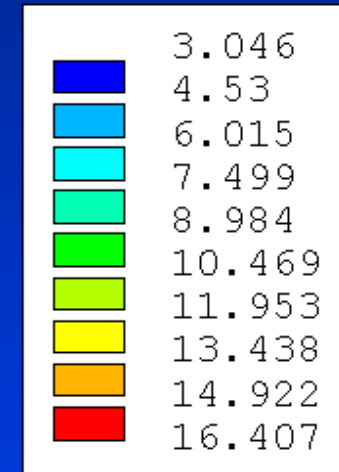
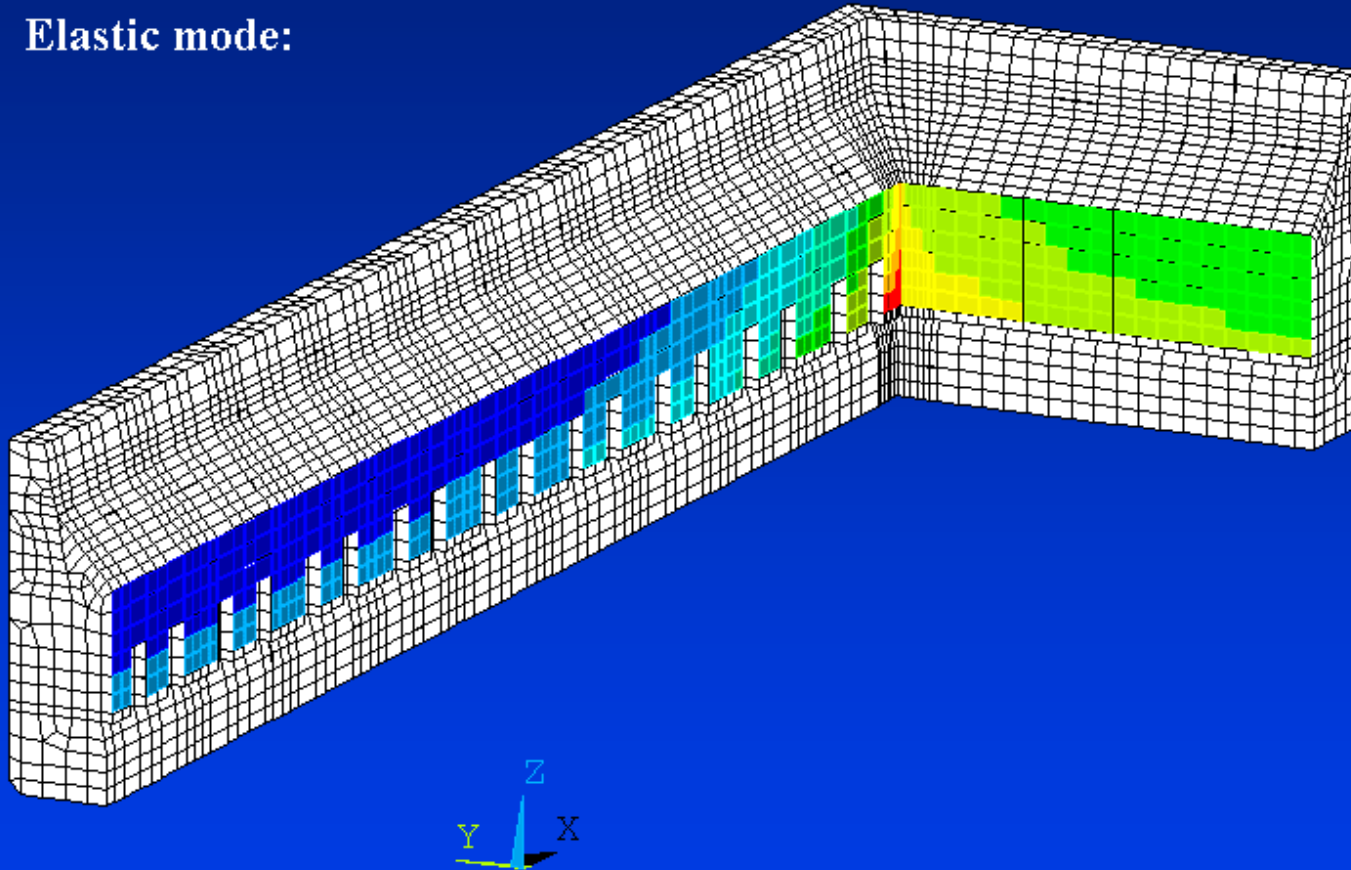
Elastic mode



A numerical scheme external to the ANSYS solver must be setup, starting from an assumed initial internal load, the task of that external numerical scheme is to converge toward that cathode block equilibrium condition pressure loading for each element face of the carbon block/side lining interface.

“Almost Empty Shell” Potshell Model

Elastic mode:

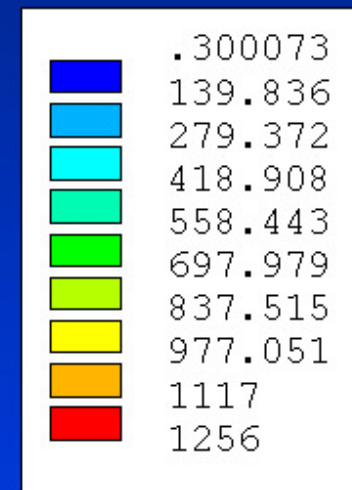
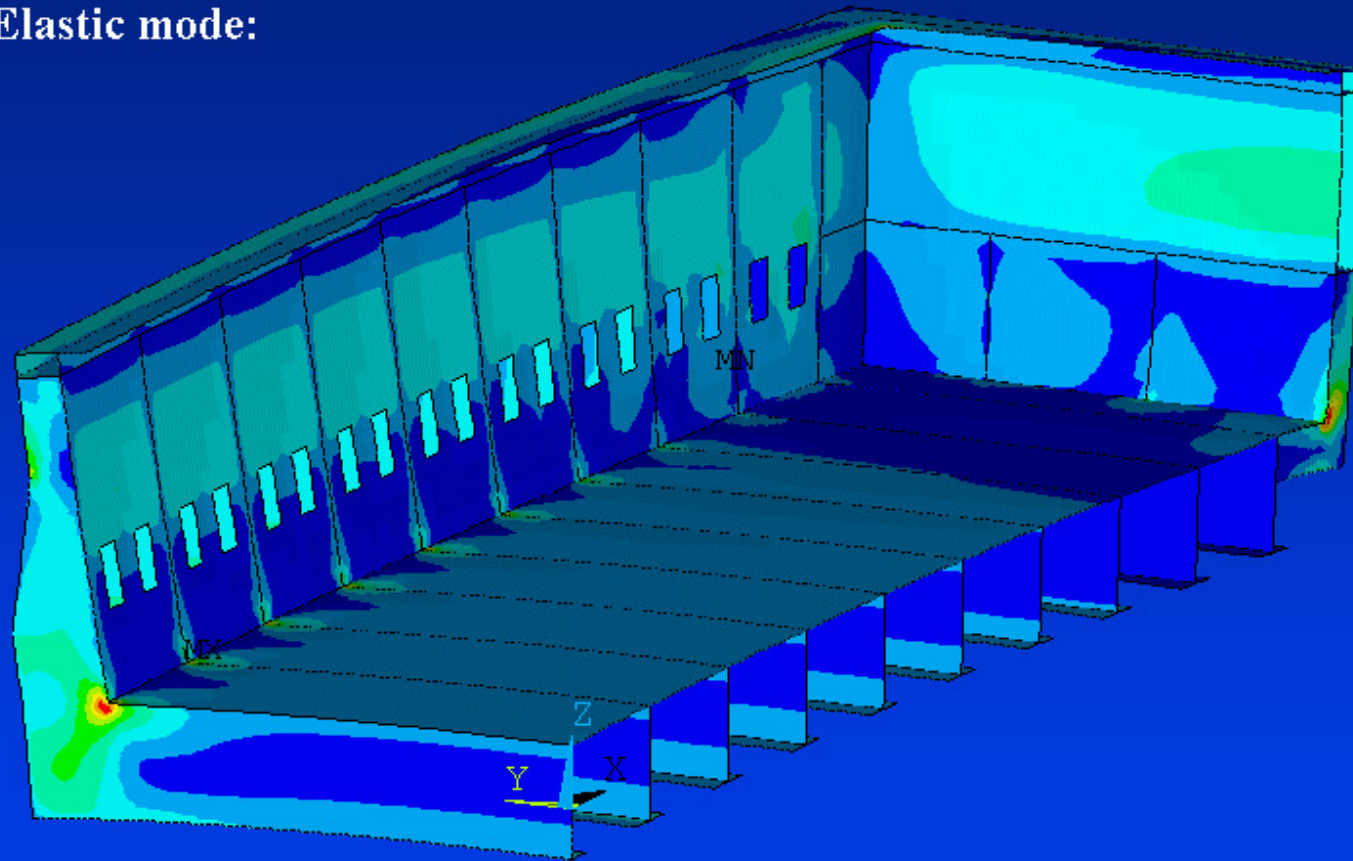


Pressure Load (MPa)

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“Almost Empty Shell” Potshell Model

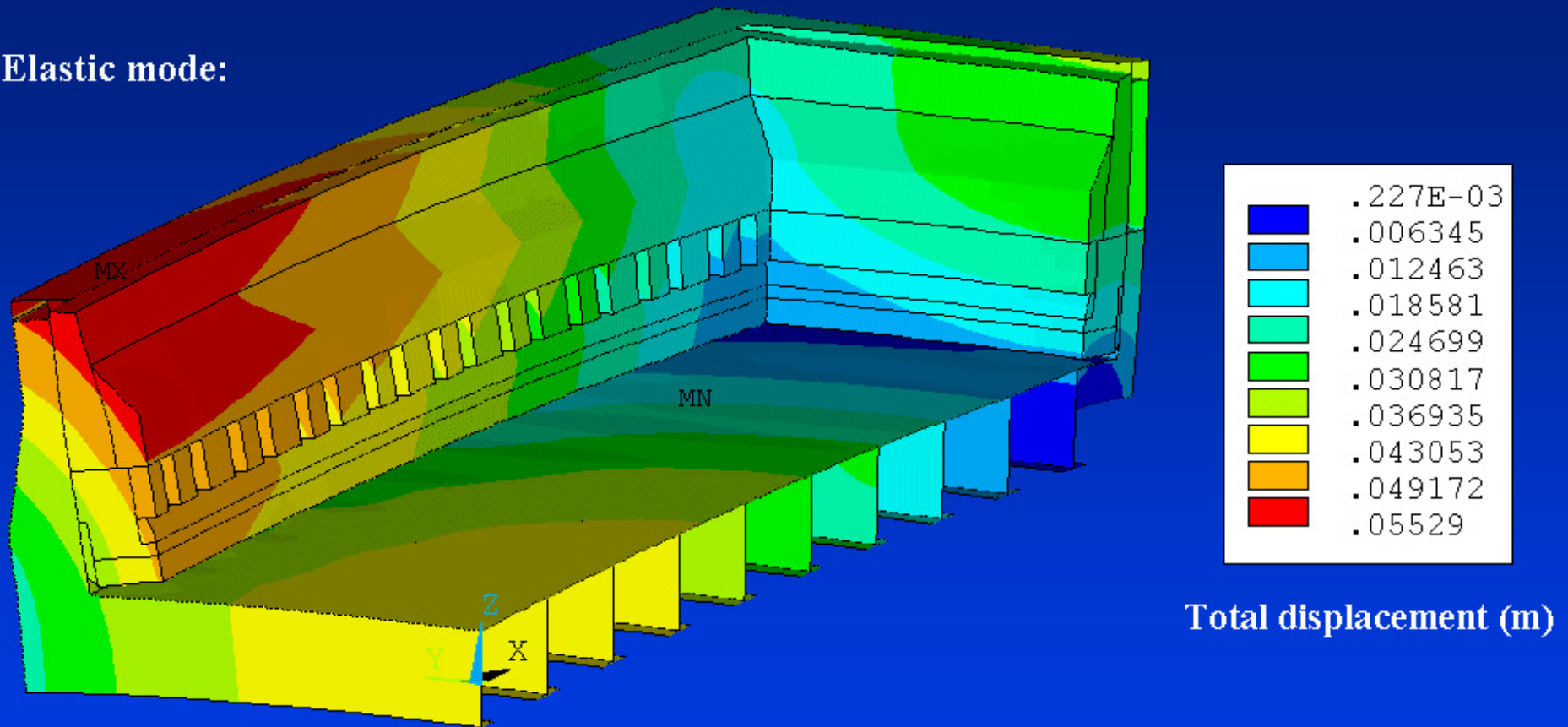
Elastic mode:



von Mises Stress (MPa)

“Almost Empty Shell” Potshell Model

Elastic mode:

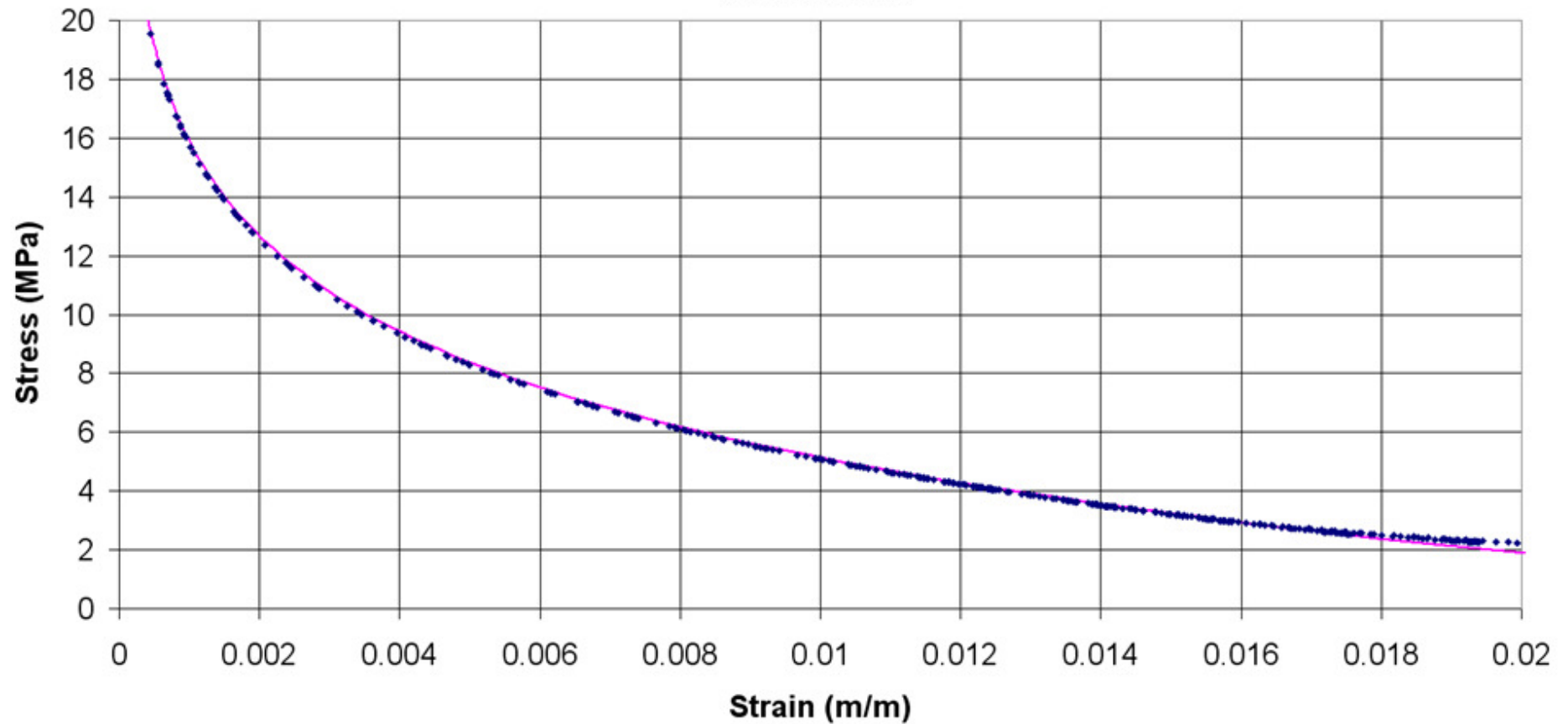


The external under-relaxation numerical scheme converges in about 20 iterations adding about the same factor to the required CPU time, in this case it took 3898 seconds CPU or 65 minutes CPU to solve.

“Almost Empty Shell” Potshell Model

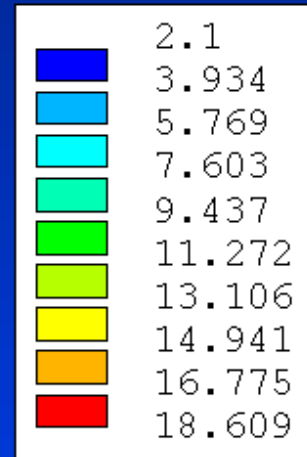
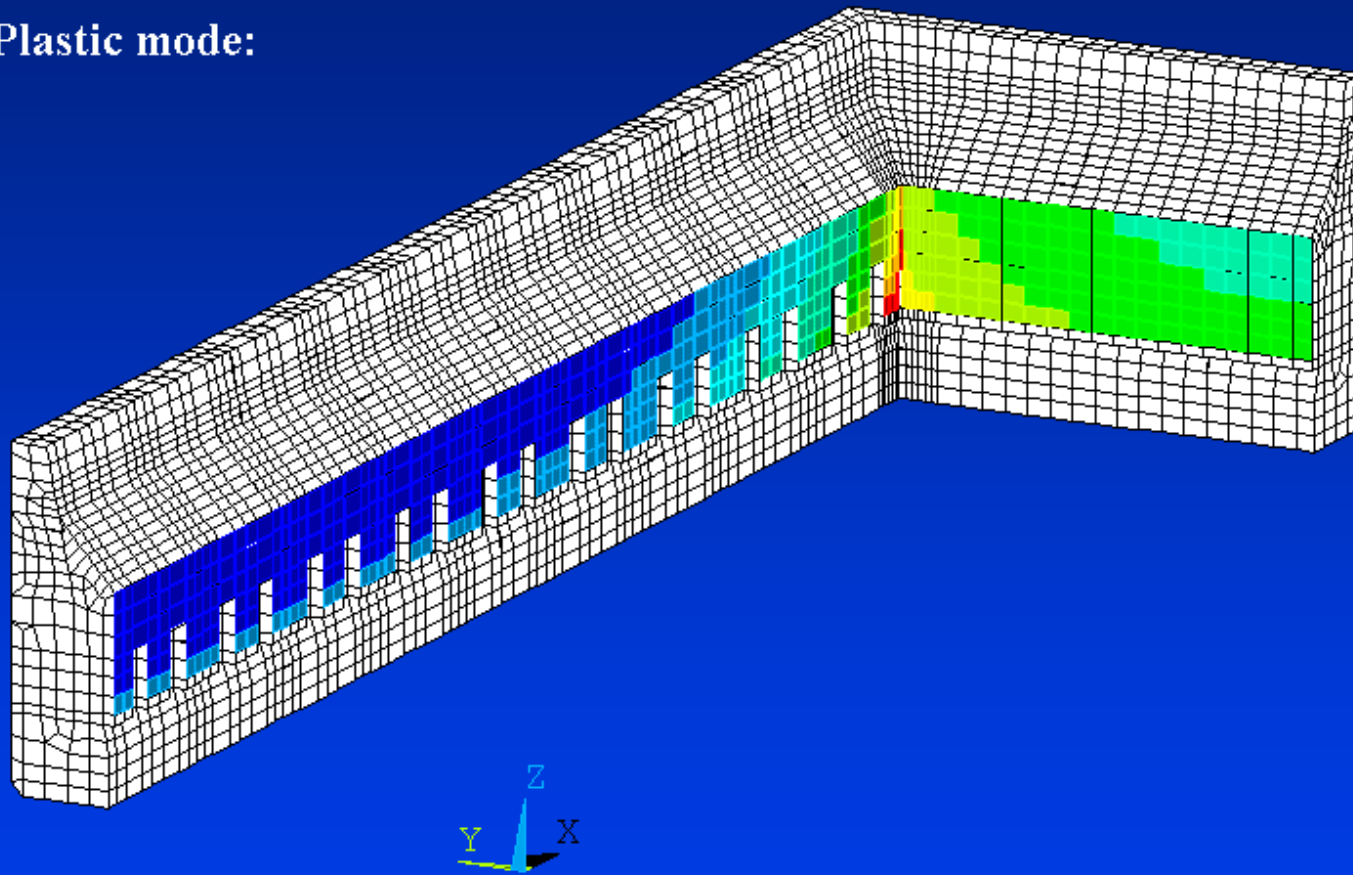
Individual element face strain-stress location on the Dewing curve

Plastic mode



“Almost Empty Shell” Potshell Model

Plastic mode:

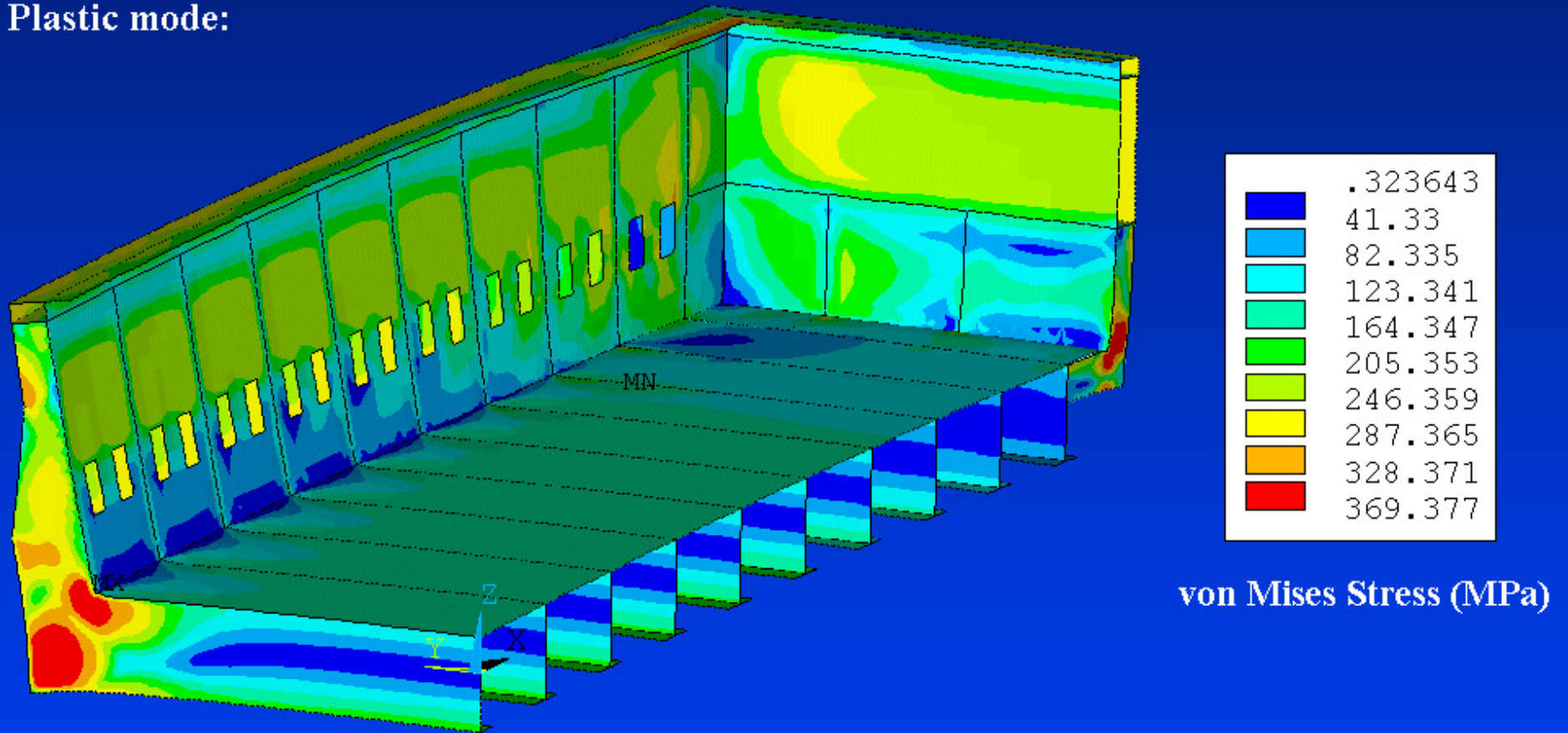


Pressure Load (MPa)

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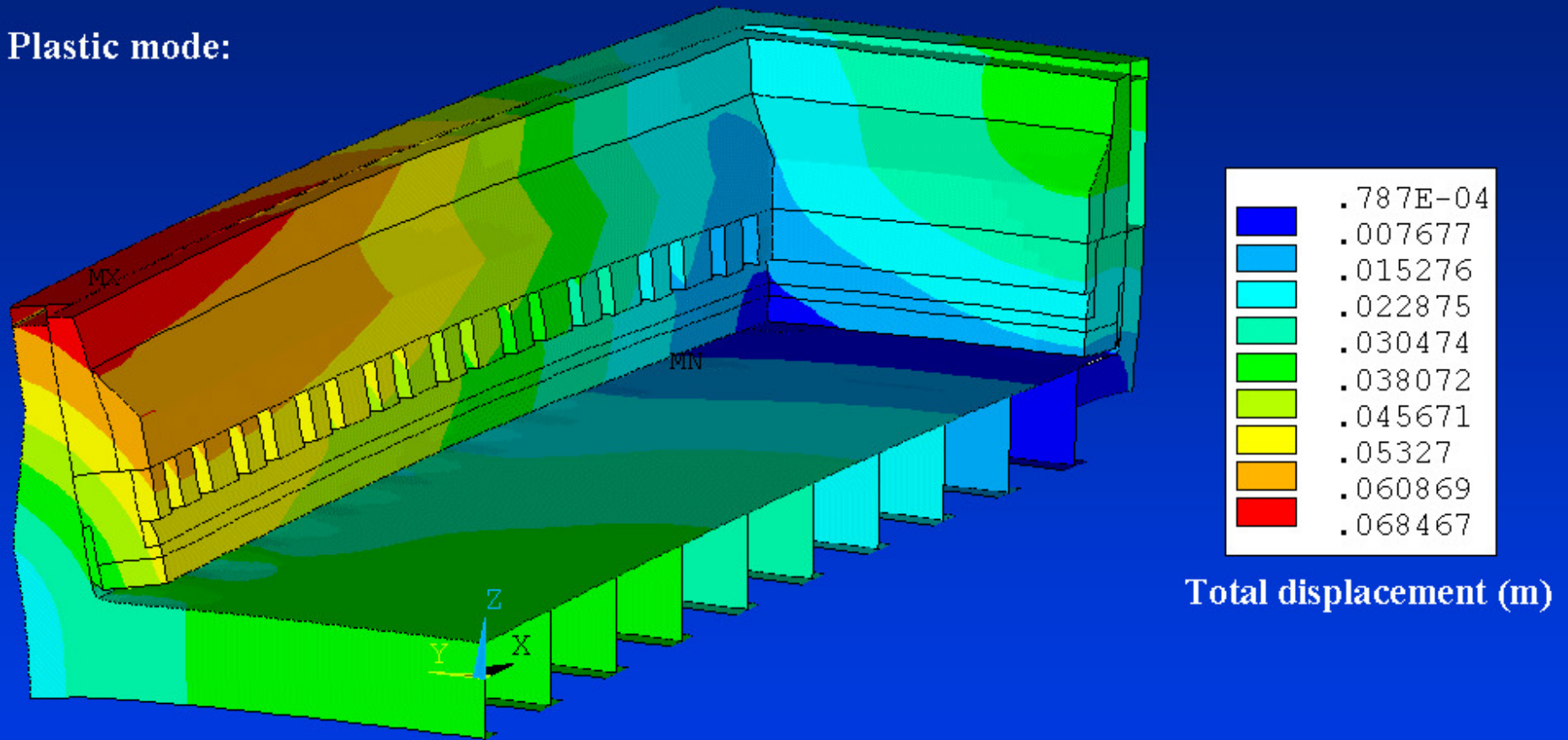
“Almost Empty Shell” Potshell Model

Plastic mode:



“Almost Empty Shell” Potshell Model

Plastic mode:



Took 27524 seconds CPU or 7.6 hours CPU to solve.

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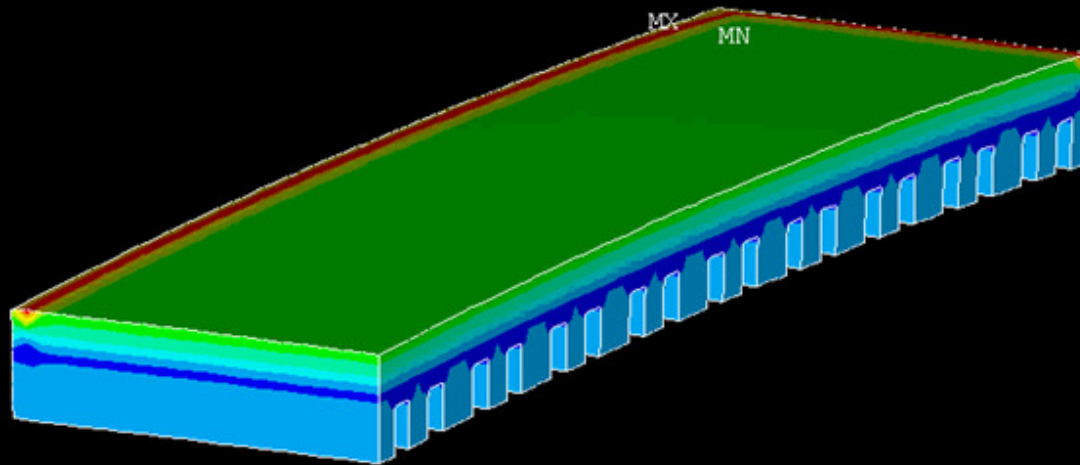
“Half Empty Shell” Potshell Model

- Based on the usage of the quadrilateral Finite Strain shell element (SHELL181) in the commercial code ANSYS®.
- Temperature distribution obtained from the full cell quarter thermo-electric model applied as a body load to the entire potshell structure.
- The geometry the lining material located between the potshell walls and the cathode carbon of the cathode blocks themselves is also included, the Dewing sodium expansion behavior of the cathode blocks is treated in ANSYS® as a “creep-like” behavior.
- Elastic mode: solving the mechanical problem only by considering only the elastic properties of the potshell steel.
- Plastic mode: solving the temperature dependent isotropic hardening von Mises plasticity behavior of the potshell steel structure using the MISO non-linear hardening option in ANSYS®.

“Half Empty Shell” Potshell Model

1

Elastic mode: carbon block strain %



```
ANSYS 11.0SP1
AUG 28 2009
09:11:47
PLOT NO. 14
NODAL SOLUTION
STEP=1
SUB =1
TIME=1
BFETEMP (AVG)
MIDDLE
DMX =.031769
SMN =-.124713
SMX =.833296
-.124713
-.018267
.088178
.194623
.301069
.407514
.51396
.620405
.72685
.833296
```

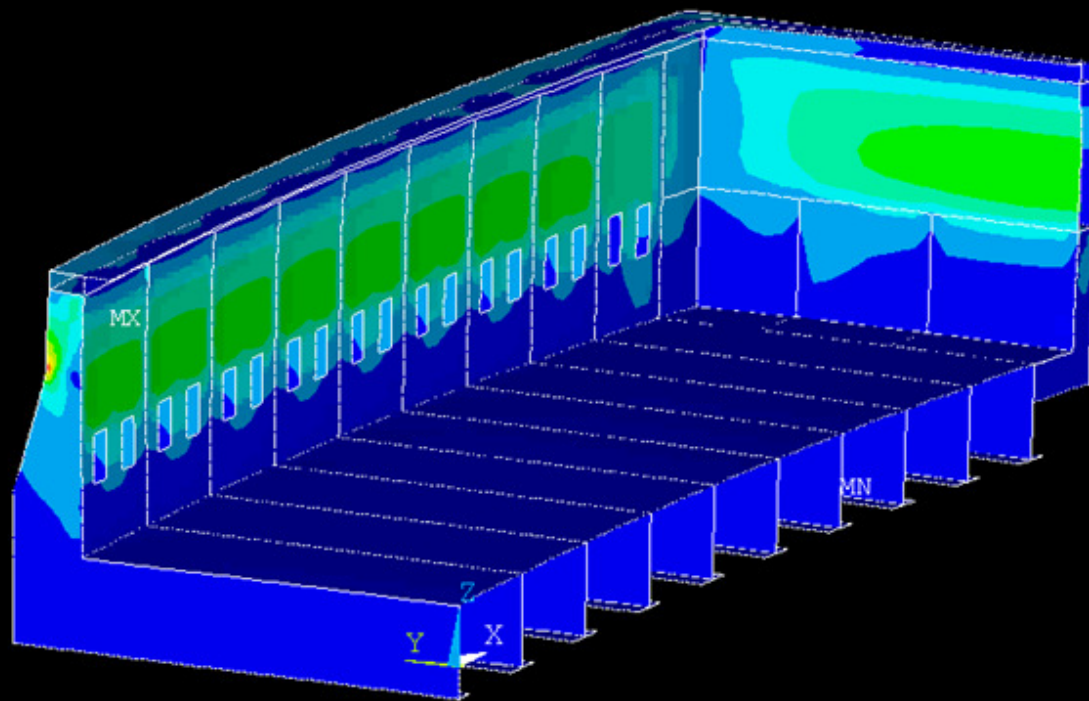
By definition, this means that it is required to solve that model in transient mode following the build-up of the sodium concentration in the cathode blocks from start-up to around 1000 days of operation where the cathode blocks get fully saturated in sodium.

Time = 0.44, maxdiff = 7.490991091E-02

“Half Empty Shell” Potshell Model

1

Elastic mode: shell stress MPa

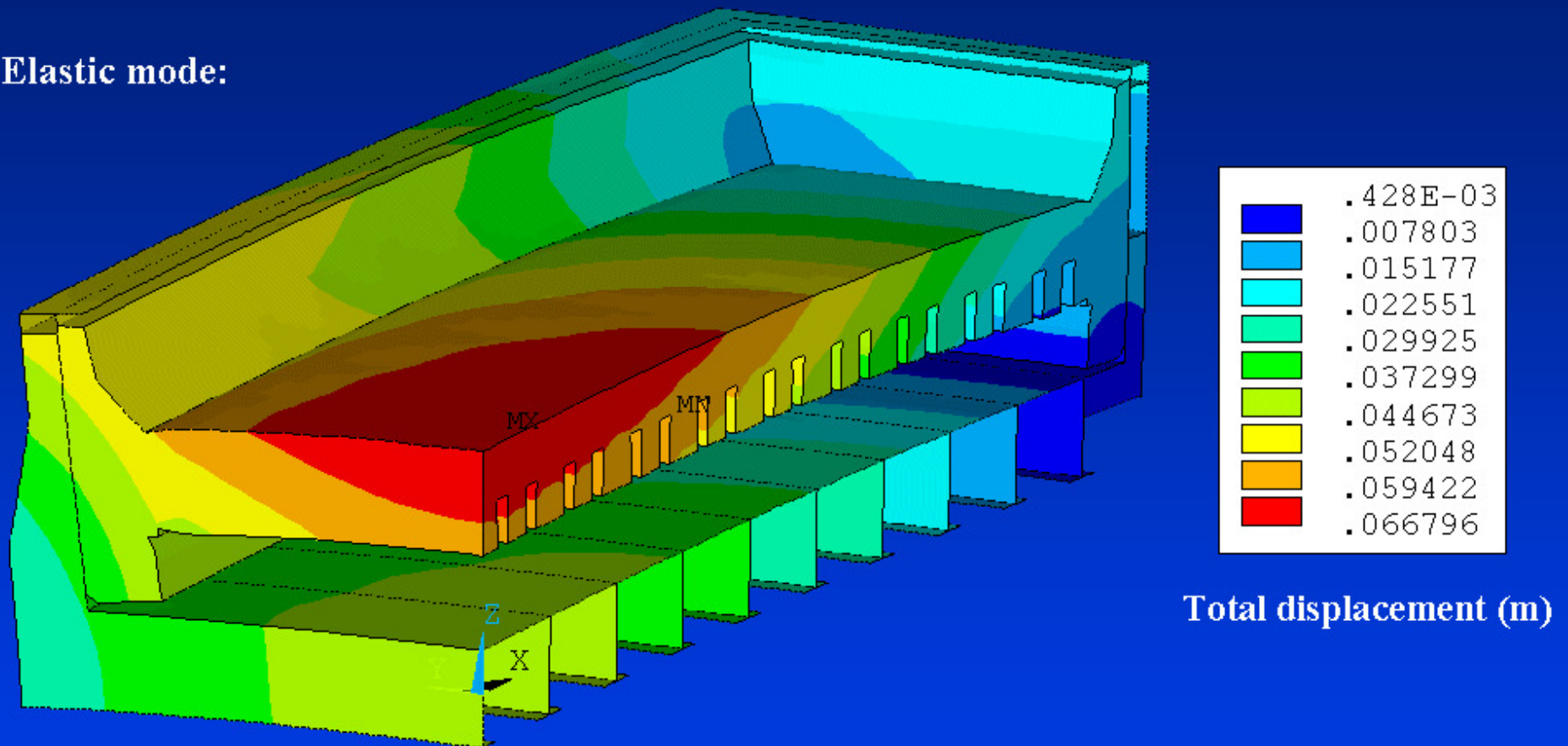


ANSYS 11.0SP1
AUG 28 2009
09:11:49
PLOT NO. 16
NODAL SOLUTION
STEP=1
SUB =1
TIME=1
SEQV (AVG)
MIDDLE
DMX =.032795
SMN =.045222
SMX =1006
.045222
111.785
223.525
335.266
447.006
558.746
670.486
782.226
893.966
1006

Time = 0.44, maxdiff = 7.490991091E-02

“Half Empty Shell” Potshell Model

Elastic mode:

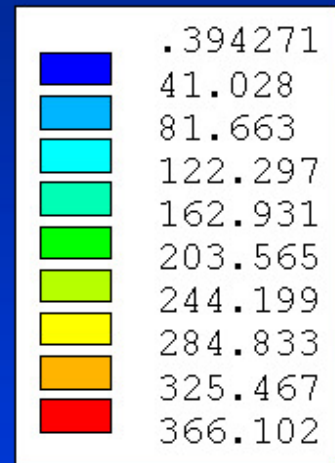
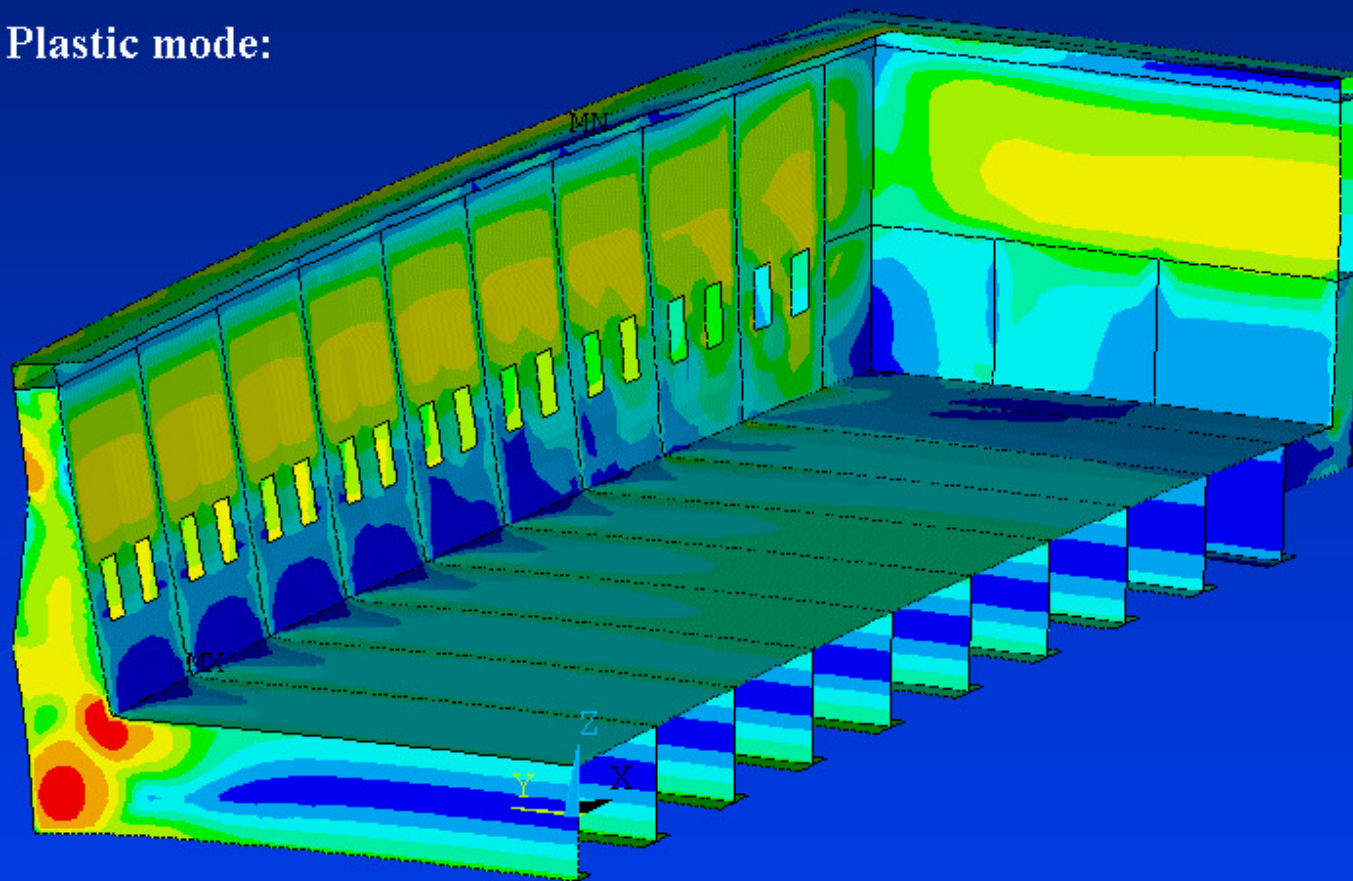


Solving the “half empty shell” potshell model in elastic properties mode took 25,335 CPU seconds or 7.0 CPU hours which is 6.5 times more than what was required to solve the “almost empty shell” potshell model.

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“Half Empty Shell” Potshell Model

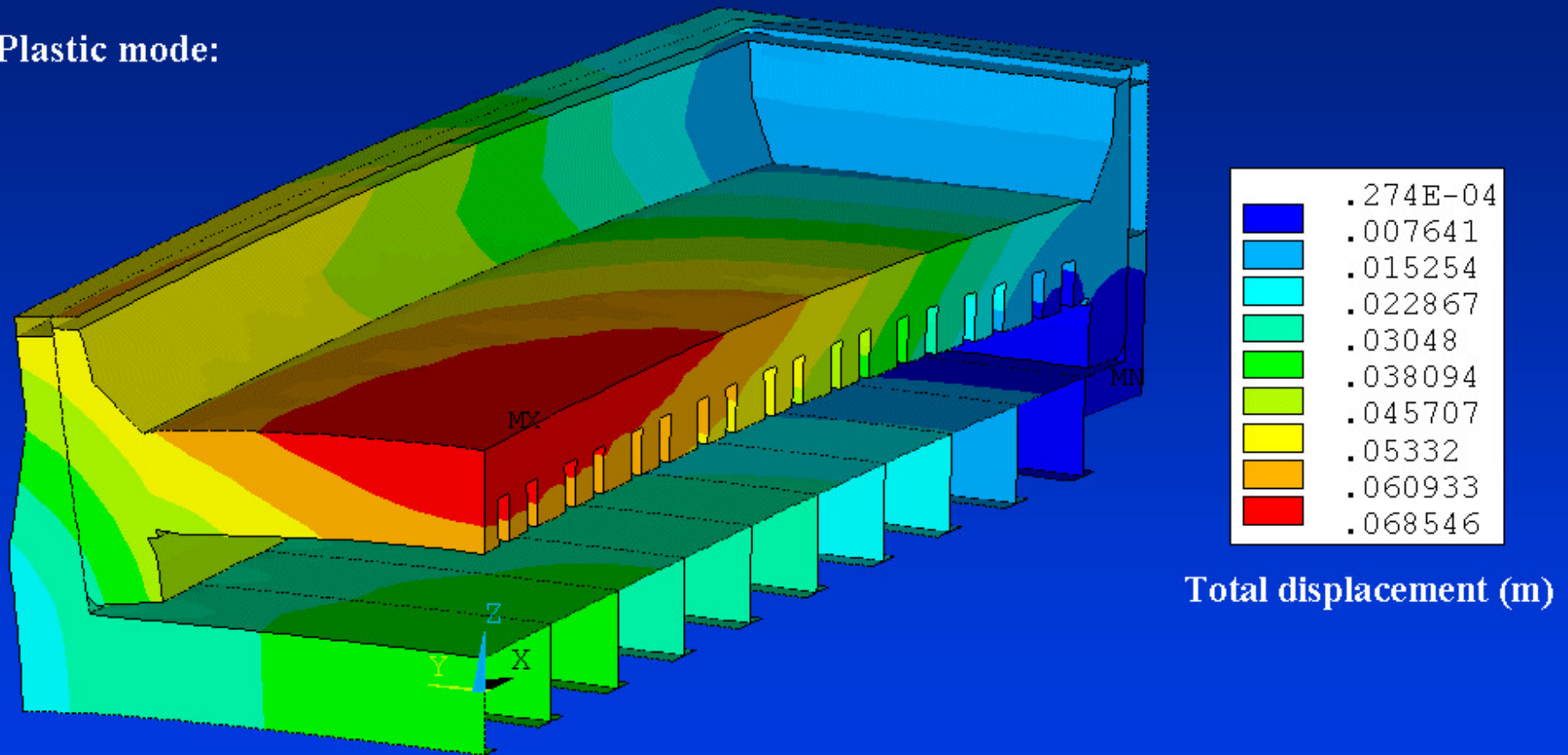
Plastic mode:



von Mises Stress (MPa)

“Half Empty Shell” Potshell Model

Plastic mode:



Took 103842 CPU seconds or 1.2 CPU days which is 3.8 times more than what was required to solve the “almost empty shell” potshell model.

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Conclusions

- Three types of ANSYS® based thermo-mechanical potshell models have been presented and described in details.
- The “empty shell” potshell model type has by far the fastest turn around time which is very convenient to be able to quickly investigate potshell design alternatives, but its ability to make accurate predictions outside of the narrow range of conditions where it has been validated is very questionable.
- The “almost empty shell” potshell model type is offering a much wider applicability range as it will automatically adjust the internal load as function of the grade of cathode blocks selected and of the rigidity of the potshell structure, it is offering an overnight turn-around time for a very little loss of accuracy.
- The “half empty shell” potshell model type is the most accurate model type of the three presented because it incorporates all the known physics of the cathode block sodium expansion phenomena affecting the potshell deformation in addition to the thermal expansion, unfortunately its turn-around time of 1.2 CPU days is already exceeding the overnight turn-around time that could be considered as the practical upper limit for an efficient design tool.

Conclusions

- **Finally, until the day where the turn-around time of the “half empty shell” potshell model type would be reduced to mere minutes as it will become maybe 20 years from now (after all it was 80 days on a CRAY X-MP/24 supercomputer 20 years ago), it could be argued that all three types of potshell models have their places in the potshell designer toolbox as sometimes turn-around time matter more than accuracy and sometimes not.**

Acknowledgements

The author wishes to thank Dr Daniel Richard of Hatch and Mr Lalit Mishra of Dubal for their much needed assistance in using ANSYS® 11.0 thermo-mechanical features, especially in plastic properties mode.